

1.11. The Trouble with the Informal Test

“...that which has been inconceivable to-day has often turned out indisputable on the morrow. Inability to conceive is only a stage through which every man must pass in regard to a number of beliefs....”

– Charles Sanders Peirce, **The Doctrine of Necessity Examined** (1892)

“In showing arguments invalid it is often necessary to determine the truth or falsehood of English sentences of a rather complex structure. Intuition is not in all cases an unequivocal guide.”

– Donald Kalish and Richard Montague, **Logic: Techniques of Formal Reasoning** (1964), p. 170

The informal test of validity can be summed up like this: using both facts about the world and the imagination, try to discover a validity counterexample for a given argument. If such a search is successful and a counterexample is found, the argument's invalid.

When introducing this test I said it was a rough, limited test of validity. Truth be told, it's really only **half** a test.

Here's why: if investigation of the actual world shows that the premises of the argument are true while the conclusion is false, then the actual world is itself a validity counterexample for the argument in question.

But even if the actual world doesn't offer up a validity counterexample, the argument can still be proven invalid so long as some other possible situation is found which qualifies as a counterexample. In that case we appeal to the imagination: if imaginatively scouring the sea of possibilities yields a hit – a validity counterexample for the argument in question – that too suffices to prove the argument invalid.

The trouble lies in cases where we draw a blank: when, after scouring the actual world and the conceivable possibilities, we fail to discover any counterexamples for the argument in question. **Should failure to think up a validity counterexample assure us that the argument is valid?**

There are good reasons to answer **no** here. Humans have a long history of misjudging the possible – specifically, of dismissing as impossible options later accepted as genuine possibilities. From science to politics, medicine to mathematics, we recognize as real possibilities what our ancestors didn't see when thinking through the very same issues. As concerns validity, that translates into an aptitude for overlooking counterexamples. Perhaps there is a validity counterexample lurking out in the sea of possibilities, but one we wrongly judge to be impossible.

A second (though related) worry also arises here: even if we recognize a situation as possible, we still might not recognize it as a counterexample for the argument we're evaluating. Perhaps that doesn't seem like a serious worry, since recognizing a situation as a validity counterexample for an argument looks simple enough: just check and see whether each of the argument's premises are true in that situation, and its conclusion false there.

Yet even within the comfortable confines of our native language, where our grasp of meanings is at its most intuitive and natural, we face the problem of **complexity**: our minds can quickly become confused in the face of complex sentence (and argument) structure.

Note that we've so far sought validity counterexamples only for arguments of the simplest sort, with a small number of uncomplicated sentences. In that case our bare grasp of English meanings was sufficient to think up a counterexample, if there was one.

Not so with the following complex, dessert-enhanced argument.¹

1. If we have either ice cream or cake, then either we'll have ice cream without having pie or we'll have both brownies and sherbet.
2. We'll have cake and brownies but we won't have both pie and fudge.
3. Unless we have pie without having fudge, we'll have neither brownies nor sherbet.

∴ Either we'll have sherbet without having ice cream, or we'll have fudge without having ice cream.

¹ See 3.5.1 Problem C33.

Imagine, for instance, a situation where we have every dessert listed here except fudge. That certainly seems possible. But does that situation qualify as a validity counterexample for this argument? Even armed with a fine knowledge of English, we draw a blank.

We certainly understand each word, and even longer stretches of words taken in isolation. But moving from trees to forest, we're stumped: it's difficult to understand exactly what claim is being staked by any one of those premises – and all the more so taking all three together along with the conclusion. And since we typically need to understand a sentence before we can tell if it's true or false in a given situation², we're equally at a loss to measure this situation against the above argument and decide if it qualifies as a validity counterexample.

Our limited ability to recognize genuine possibilities, and our tendency to be **boggled by complexity** even in the familiar terrain of our own native language, undercuts the informal, imagination-based search for validity counterexamples. Our original question was: if we try and fail to imagine a counterexample for an argument, what does this tell us about the argument's validity? We see now that if the argument is at all complicated, drawing such a blank really tells us **nothing**: our imaginative search may have failed because there really are no counterexamples, or because counterexamples exist but we couldn't recognize them as possible, or our mental resources were overwhelmed by complexity. That means the informal, imagination-based test of validity will at best work only in simple, familiar cases. More difficult examples call for a more robust test which can be trusted to survey all the possibilities, and which isn't boggled by complexity.

The situation here is roughly analogous to our grasp of arithmetic. For basic sums – $2+3$, $5+1$, or $6+4$ – we can see the correct answer in a simple mental glance. But for more difficult sums – say, $348,297 + 862,378$ – we have no such intuitions. In those cases we give up trying to juggle figures in our heads, resorting instead to the familiar mechanical procedure of summing one column, carrying digits to the next column, and repeating as necessary.

By its nature such a mechanical procedure is not so intuitive. But given the fragility of our intuitions in the face of complexity, we now see that's not entirely a bad thing.

² As noted in 1.X.

Our test of validity will follow a similar course in later chapters – replacing simple intuitive judgments of the unaided intellect and imagination with more mechanical but resilient methods. Those methods take us beyond informal logic, into the domain of **formal logic**.